Versatility of 'continuations' in discourse semantics

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Abstract. We show in this paper how the computer science concept of 'continuations', together with categorial grammars and a type shifting mechanism, is able to account for a wide range of natural language semantic phenomena, such as hierarchical discourse structure, ellipses, accommodation and free-focus and bound-focus anaphora. The merit of continuations in the dynamic semantics framework is that they abstract away from assignment functions that are essential to the formulations of Dynamic Intensional Logic, Dynamic Montague Grammar, Dynamic Predicate Logic and Discourse Representation Theory. Thus, continuation style semantic do not pose problems such as the destructive assignment problem in Dynamic Predicate Logic or the variable clash problem in Discourse Representation Theory. We argue that continuations are a versatile and powerful tool, particularly well suited to manipulate scope and long distance dependencies, phenomena that abound in natural language semantics.

Keywords: 'continuations', dynamic semantics, lambda calculus, type shifting, scope, anaphora.

1. Introduction

We show in this paper how the computer science concept of 'continuations', together with categorial grammars and a type shifting mechanism, is able to account for a wide range of natural language semantic phenomena, such as hierarchical discourse structure, ellipses, accommodation and free-focus and bound-focus anaphora. We argue that continuations are a versatile and powerful tool, particularly well suited to manipulate scope and long distance dependencies, phenomena that abound in natural language semantics.

Continuation have been previously used to account for intra-sentential linguistic phenomena such as focus fronting, donkey anaphora, presuppositions, crossover or superiority (Barker 2002, Barker 2004, C Corresponding author
Shan 2005, Shan and Barker 2006, Barker and Shan 2008), for cross-sentential semantics in (de Groote, 2006, 2010) and for analyzing discourse structure in (Asher and Pogodalla, 2010). The merit of continuations in the dynamic semantics framework is that they abstract away from assignment functions that are essential to the formulations of Dynamic Montague Grammar (Montague, 1970), Dynamic Predicate Logic (Groenendijk and Stokhof, 1990, 1991) Discourse Representation Theory (Kamp and Reyle, 1993) and File Change Semantics (Heim, 1983). Thus, continuation style semantics do not have problems such as the destructive assignment problem in DPL or the variable clash problem in DRT. These theories deal with interpreting a discourse (giving a compositional semantics to a sequence of sentences). A key challenge in this framework is interpreting cross-sentential anaphora. Assigning a first order logical representation to a discourse like A man came. He whistled. is problematic. How can we get from the two first order representations $\exists x. (\text{man}(x) \land \text{came}(x))$ and $\text{whistled}(x)$, the representation $\exists x. ((\text{man}(x) \land \text{came}(x)) \land \text{whistled}(x))$, i.e. obtaining a bound variable from a free one? Continuations are a versatile and powerful tool, particularly well suited to manipulate scope and long distance dependencies, thus capable of representing such discourse semantic phenomena.

Continuations are a standard tool in computer science, used to control side effects of computation. They are a notoriously hard to understand notion. Actually, understanding what a continuation is per se is not so hard. What is more difficult is to understand how a grammar based on continuations (a ‘continuized’ grammar) works. The basic idea of continuizing a grammar is to provide subexpressions with direct access to their own continuations (future context), so subexpressions are modified to take a continuation as an argument. A continuized grammar is said to be written in continuation passing style and it is obtained from any grammar using a set of formal general rules. Continuation passing style is in fact a restricted (typed) form of $\lambda$-calculus. Historically, the first continuation operators were undelimited (e.g., call/cc or $J$). An undelimited continuation of an expression represents "the entire (default) future for the computation of that expression". Felleisen (1988) introduced delimited continuations (sometimes called ‘composable’ continuations) such as control (‘C’) and prompt (‘%’). Delimited continuations represent the future of the computation of the expression up to a certain boundary. Interestingly, the natural-language phenomena discussed here make use only of delimited continuations.

For instance, if we take the local context to be restricted to the sentence, when computing the meaning of the sentence John saw Mary, the default future of the value denoted by the subject is that it is destined to have the property of seeing Mary predicated of it. In symbols, the continuation of the subject denotation $j$ is the function $\lambda x. \text{saw } m x$. Similarly, the default future of the object denotation $m$ is the property of being seen by John, the function $\lambda y. \text{saw } y \ j$; the continuation of the transitive verb denotation $\text{saw}$ is the function $\lambda R. R \ m \ j$; and the continuation of the VP $\text{saw Mary}$ is the function $\lambda P. P \ j$. This simple example illustrates two important aspects of continuations:

1. every meaningful subexpression has a continuation;
2. the continuation of an expression is always relative to some larger expression containing it.

Thus when John occurs in the sentence John left yesterday, its continuation is the property $\lambda x. \text{yesterday left } x$; when it occurs in Mary thought John left, its continuation is the property $\lambda x. \text{thought(} \text{left } x \text{)} \ m$ and when it occurs in the sentence Mary or John left, its continuation is $\lambda x. (\text{left } m) \lor (\text{left } x)$ and so on.

Further on, the paper is structured as follows: section 2 presents the formalism we use. In sections 3 to 6 we show how continuation-based semantics accounts for aspects of hierarchical discourse structure, ellipses, accommodation and free-focus and bound-focus anaphora. We conclude in section 7.
2. The formalism

We use Barker’s tower notation for a given expression, which consists of three levels: the top level specifies the syntactic category of the expression coached in categorial grammar (the categories act either as functions or as attributes), the middle level is the expression itself and the bottom level is the semantic value.

<table>
<thead>
<tr>
<th>syntactic category</th>
<th>expression</th>
<th>semantic value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

The syntactic categories are written $\frac{C_1 D}{A/B}$, where $A$, $B$ and $C$ can be any categories. We read this counter clockwise as "the expression functions as a category $A$ in local context, takes scope at an expression of category $B$ to form an expression of category $C".

The semantic value $\lambda k. f[k(x)]$ is equivalently written vertically as $f[x]$ omitting the future context (continuation) $k$. Here, $x$ can be any expression, and $f[]$ can be any expression with a gap $[]$. Free variables in $x$ can be bound by binders in $f[]$. This notational convention is meant to make easier (more visual) then in linear notation the combination process of two expressions. Here are the two possible modes of combination (Barker & Shan 2008):

\[
\begin{align*}
\left( \begin{array}{cc}
\frac{C_1 D}{A/B} & \frac{D_1 E}{B/A} \\
\frac{g[\ ]}{x} & \frac{h[\ ]}{f(x)} \\
\end{array} \right) & \Rightarrow \left( \begin{array}{cc}
\frac{C_1 E}{A} & \\
\frac{g[h[\ ]]}{f(x)} & \\
\end{array} \right) \\
\left( \begin{array}{cc}
\frac{C_1 D}{B/A} & \frac{D_1 E}{B/A} \\
\frac{g[\ ]}{x} & \frac{h[\ ]}{f(x)} \\
\end{array} \right) & \Rightarrow \left( \begin{array}{cc}
\frac{C_1 E}{A} & \\
\frac{g[h[\ ]]}{f(x)} & \\
\end{array} \right)
\end{align*}
\]

Below the horizontal lines, combination proceeds simply as in combinatory categorial grammar: in the syntax, $B$ combines with $A/B$ or $B\setminus A$ to form $A$; in the semantics, $x$ combines with $f$ to form $f(x)$. Above the lines is where the combination machinery for continuations kicks in. The syntax combines the two pairs of categories by a kind of cancellation: the $D$ on the left cancels with the $D$ on the right. The semantics combines the two expressions with gaps by a kind of composition: we plug $h[\ ]$ to the right into the gap of $g[\ ]$ to the left, to form $g[h[\ ]]$. The expression with a gap on the left, $g[\ ]$, always surrounds the expression with a gap on the right, $h[\ ]$, no matter which side supplies the function and which side supplies the argument below the lines. This fact expresses the generalization that the default order of semantic evaluation is left-to-right.

When there is no quantification or anaphora involved, a simple sentence like John came. is derived as follows.

\[
\begin{align*}
\left( \begin{array}{cc}
DP & DP\setminus S \\
\text{John} & \text{came} \\
\text{j} & \text{came} \\
\end{array} \right) & \Rightarrow \begin{array}{c}
S \\
\text{John came} \\
\text{came j}
\end{array}
\end{align*}
\]
In the syntactic layer, as it is usual in categorial grammar, the category under slash (here DP) cancels with the category of the argument expression; the semantics is function application.

Quantificational expressions have extra layers on top of their syntactic category and on top of their semantic value, making essential use of the powerful mechanism of continuations in ways proper names or definite descriptions do not. For example, below is the derivation for *A man came*.

\[
\begin{array}{c}
S|S/DP/N & N & S|S/DP/S \\
\lambda P, \exists x. P(x) & \text{man came} & \exists x. \text{man}(x)\\
\end{array}
\]

The comparison between the above analysis of "John came." and that of "A man came." reveals that *came* has been given two distinct values. The first, simpler value is the basic lexical entry, the more complex value being derived through the standard type-shifter Lift, proposed by Partee and Rooth (1983), Jacobson (1999), Steedman (2000), and many others:

\[
A \xrightarrow{\text{expression}} \text{Lift} \xrightarrow{B|B/A} \text{expression} \xrightarrow{\lambda k. k(x)}
\]

Syntactically, *Lift* adds a layer with arbitrary (but matching!) syntactic categories. Semantically, it adds a layer with empty brackets. In linear notation we have: \( x \xrightarrow{\text{Lift}} \lambda k. k(x) \). To derive the syntactic category and a semantic value with no horizontal line, Barker and Shan (2008) introduce the type-shifter *Lower*. In general, for any category \( A \), any value \( x \), and any semantic expression \( f[\ ] \) with a gap, the following type-shifter is available.

\[
A \xrightarrow{\text{expression}} \text{Lower} \xrightarrow{f[\ ]} \text{expression} \xrightarrow{f[x]}
\]

Syntactically, *Lower* cancels an \( S \) above the line to the right with an \( S \) below the line. Semantically, *Lower* collapses a two-level meaning into a single level by plugging the value \( x \) below the line into the gap \([\ ]\) in the expression \( f[\ ] \) above the line. *Lower* is equivalent to identity function application.

The third and the last type shifter we need is the one that treats binding. Binding is a term used both in logics and in linguistics with analog (but not identical) meaning. In logics, a variable is said to be bound by an operator (as the universal or existential operators) if the variable is inside the scope of the operator. If a variable is not in the scope of any operator, than the variable is said to be free. In linguistics, a binder may be a constituent such as a proper name (*John*), an indefinite common noun (*a book*), an event or a situation, or even a plural common noun (*books*). Anaphoric expressions such as pronouns (*he, she, it, him, himself, etc*), definite common nouns (*the book, the book that John read*), demonstrative pronouns (like *this, that*), ellipses, etc act as variables that take the value of (are bound by) a previous binder. We adopt the idea (in line with (Barker and Shan (2008))) that the mechanism of binding is the same as the mechanism of scope taking.

In order to properly account for binding, we need to formulate an explicit semantics for both the binder and the anaphoric expressions to be bound. Barker and Shan (2008) explicitly state the Binding
rule for determiner phrases (DPs) that may act as binders:

\[
\frac{A|B}{\text{expression}} \quad \frac{A|DP\triangleright B}{\text{expression}} \quad \frac{f[y]}{x} = f[x]
\]  

(7)

At the syntactic level, the Bind rule says that an expression that functions in local context as a DP may look to the right to bind an anaphoric expression (Barker and Shan encode that by the sign \(\triangleright\)). At the semantic level, the expression transmits (copies) the value of the variable \(x\). In linear notation, the semantic part of the Bind rule looks like: \(\lambda k.f[k(x)]\triangleright\lambda k.f([k(x)]x)\).

As for the elements that may be bound, Barker and Shan give the following lexical entry for singular pronouns (\(he, she, it\)):

\[
\frac{DP|S|S}{\text{expression}} \quad \frac{he}{\lambda y.y} \quad \frac{f[y]}{x}
\]  

(8)

To account for multiple anaphoric expressions (and their binders) or for inverse scope of multiple quantifiers, each binder can occupy a different scope-taking level in the compositional tower. With access to multiple levels, it is easy to handle multiple binders. Then, a pronoun or another anaphoric expression chooses its binder by choosing where to take scope. So, distinct scope-taking levels correspond to different binders, layers playing the role of indices: a binder and the anaphoric expression it binds must take effect at the same layer in the compositional tower. A superior level takes scope at inferior levels and left expressions take scope at right expressions, to account for left-to-right natural language order of processing.

We propose here the semantics of cross-sentential punctuation marks ‘.’ and ‘;’, thus treating only assertive (affirmative or negative) sentences. We consider ‘.’ and ‘;’ as functions that take two sentence denotations and return a sentence denotation (the conjunction of original sentence denotation):

\[
\begin{align*}
S\langle(S/S) \quad S\langle(S/S) \quad 
\cdot \quad 
\lambda p \lambda q.p \land q \quad \lambda p \lambda q.p \land q
\end{align*}
\]  

(9)

For two affirmative sentences with no anaphoric relations and no quantifiers, such as John came. Mary left., the derivation trivially proceeds as follows:

\[
\begin{align*}
S \quad S\langle(S/S) \quad S \quad S
\cdot \quad 
\text{John came} \quad \text{Mary left} = \text{John came. Mary left}
\end{align*}
\]  

\[
\begin{align*}
\text{came j} \quad \lambda p \lambda q.p \land q \quad \text{left m} \quad \text{came j \land left m}
\end{align*}
\]  

(10)

As one sees above, there is no need in this simple case to resort to type shifting at all. Nevertheless, type shifting and the powerful mechanism of continuations are employed when dealing with linguistic side
effects such as quantifier scope or binding. Thus, to transmit quantificational or anaphoric information, the denotation lifts accordingly, for example like this:

$$S|S \quad D_P > S|D_P > S$$

or

$$S(S/S) \quad S(S/S)$$

(11)

To derive the denotation of *A man came. He whistled.*, type lifting, type lowering and the Bind rule become necessary.

$$S|S \quad S|S$$

$$\lambda x. (\lambda y. \lambda P. x : S\)$$

$$\lambda x. (\lambda y. \lambda P. x : S\)$$

(12)

(13)

(14)

(15)

(16)

Note that the denotations of *came* and *whistled* were also lifted so as to match the ones of *a* and *he*, both being scope-takers. The last equality sign is due to routine lambda conversion.

3. Handling Hierarchical Discourse Structure

While at the sentence level the combination of elements proceeds via syntactic rules, at the discourse level the combination of sentences proceeds via rules imposed by rhetorical relations between sentences. This parallelism functions at semantic level too: syntax determines how the meaning of words should be
put together to provide the meaning of the whole sentence; analogously, the meaning of the sentences and the way they are combined via rhetorical relations determines the meaning of the discourse.

Rhetorical relation theory is introduced in Mann and Thompson (1986). We will refer to Asher and Lascarides (2003) for more recent and extensive theory that combines the dynamic semantics (Discourse representation theory style) with discourse structure via rhetorical relations. We only mention here the way of choosing a distinct rhetorical relation: R is a relation iff there is evidence that it affects truth-conditions of the sentences it connects, which cannot be explained by other means. For instance, Asher and Lascarides (2003) give the following rhetorical relations:

- **Coordinating relations** (the two constituents they relate have the same importance in the discourse): Narration, Background, etc.

- **Subordinating relations** (the second constituent is not essential for the discourse and it may be dropped): Elaboration, Explication, Result;

- **Structural**: Contrast, Parallel.

We are interested here neither in formulating explicit rules to combine sentences via rhetorical relation, nor in giving an exhaustive treatment of the phenomena (see Asher and Lascarides 2003). Instead, we will give enough details to make plausible the use of continuations for modelling hierarchical discourse structure. Specifically, we will treat some side effects of rhetorical relations, such as the constraints that hierarchical structure (induced by subordinating rhetorical relations) imposes on the interpretation of pronominal anaphora.

There are two types of side effects: local and global. In the case of local effects, the rhetorical relation between a sentence \( A \) and a sentence \( B \) constrains the interpretation of pronouns that occur in \( B \) (Kehler 2002). In the case of global effects, the relation between \( A \) and \( B \) (be it a clause, a sentence, a phrase or a chunk of discourse) constrains the interpretation of pronouns that occur in some subsequent clause \( C \). This is known as the Right Frontier constraint (Polanyi 1985): the antecedent of a pronoun in the current sentence must be introduced by the previous utterance or one that dominates it in the discourse structure. To exemplify the Right Frontier Constraint, we give an example from Asher and Lascarides (2003):

\[
\begin{align*}
John & \text{ had a great evening.} \\
He & \text{ had a great meal.} \\
He & \text{ ate salmon.} \\
He & \text{ devoured cheese.} \\
He & \text{ then won a dance competition.} \\
*It & \text{ was a beautiful pink.}
\end{align*}
\]

This discourse has the structure in figure 1.

Elaboration is a subordination relation, so it introduces a new (subordinating) level represented on the vertical. Narration is a coordinating relation, thus the two components remain on the same (horizontal) level. The impossibility that \( it \) refers back to \( salmon \) is accounted for by the Right Frontier constraint: the antecedent \( salmon \) is not in the previous sentence or upper in the structure of discourse, thus it is inaccessible for subsequent reference.

To account for RFC, we will formulate a naïve interpretation for the relation of subordination (be it Explanation, Elaboration, etc). It is naïve in two respects: first, it ignores other rhetorical relation-specific aspects of interpretation; second, we will only consider the point separator between sentences,
that leaves implicit the exact rhetorical relation between the sentences it relates and which is, by no means, the only one that introduces rhetorical relations: consider for instance explicit lexical entries like but, also, on the one hand, on the other, so, because, when, while, nevertheless, etc. When “.” introduces a new subordinated piece of discourse, we take its lexical entry to be:

\[ S\backslash\left(\frac{S|S}{S}\right) \]

\[ \lambda p\lambda q. p^\wedge \frac{q}{q} \] (17)

At the syntactic level, subordinating point takes two components of type S (the first one is the previous discourse) and gives a component of type \( S|S \). At the semantic level, the subordinating point introduces a new level on which it places the previous discourse and a gap; the gap is just a place holder for the subordinated future discourse. When the subordinated discourse ends, one returns upper in the discourse structure by applying Lower, which plugs all subordinated discourses into the gap and collapses the two levels introduced by subordinating point. In doing so, the scope of all DPs that could have bound a subsequent pronoun also closes, making further reference to them impossible. This accounts for RFC and thus for the impossibility of the pronoun it to refer back to the antecedent salmon in the upper example.

4. Ellipsis

Since ellipsis acts as an anaphora (it picks out a previously introduced constituent), we expect it to function in discourse similarly to other anaphora (definite descriptions or pronouns). Thus, ellipsis looks left for an expression to bind (transmit its value to) it. Depending on their category, there are several types of ellipses, for instance noun phrase ellipses or verb phrase ellipses. We will only sketch here the treatment of nominal ellipses. The other types may be treated similarly.

In English, Noun Phrase ellipsis is only possible for plurals. The problem is what exactly gets elided. Consider the two examples of cross-sentential nominal ellipsis (Cornilescu, 2011):

---

\(^1\)In Romanian, singular NPE is possible, but only because unarticulated singular nouns (such as masina-car) denote a kind in Romanian. For instance:
1. Some men entered. Some (men) remained out.

2. Some men entered. Most (of the men who entered) sat dawn.

In the first case, what gets elided is the common noun men previously introduced as the restrictor of the first quantificational determiner some. In the second case, what gets elided is the whole plural referent previously introduced, i.e. the intersection of the restrictor and nuclear scope of the quantificational determiner some. While in the first example the quantificational determiner some should take as argument a plural common noun (men) at the ellipsis site, in the second example, the quantificational determiner most requires as argument at ellipsis site a determiner phrase preceded by of (of the man who came). This syntactic difference implies a semantic difference. We will first treat the (syntactically simpler) case of plural quantificational determiners that take as arguments a common noun (without preposition of). We assume that what gets elided is the plural common noun previously introduced as the restriction of a DP antecedent. Observe that, although there is a mathematical difference between a set (plural individual) and a predicate, one may always consider a set as a predicate true of all the objects in the set and false of all the others. Our proposal is that on the one hand, the plural common noun introduced as the restriction of a DP antecedent offers to bind the ellipsis site and on the other, the ellipsis site is filled with a silent lexical entry that functions in local context as a plural common noun, takes scope at a sentence to make a sentence that looks left for a plural common noun (a binder):

\[
\frac{N^{pl} \rightarrow S | S}{\phi}
\]

We extend the Bind rule accordingly, to allow categories, other than DPs, to bind (transmit their values), in this case, \(N^{pl}\):

\[
\frac{A | B}{N^{pl}} \quad expression \quad Bind \quad expression \quad \frac{A | N^{pl} \rightarrow B}{N^{pl}}
\]

We take the lexical entry for the plural quantificational determiner some to be (ignoring here the maximality requirement for binding):

\[
\lambda P. \exists X. ((X \geq 1 \wedge P(X)) \wedge (\_))
\]

Ion vrea masina rosie. Maria vrea [masina]e galbena.
Ion wants car red. Maria wants [car]e yellow.
It is easy to extend our theory to treat in a similar manner this kind of ellipsis in Romanian, by allowing singular Ns to bind an ellipsis site.
We have to force the scope closing of the above variable \( X \) in the usual way by applying \( \text{Lower} \), in order to prevent it to bind subsequent anaphora (transmit a non-maximal value). Here is the derivation of "Some men entered. Some (men) remained out."

\[
\begin{align*}
\frac{\text{NP} \downarrow S | S}{\text{NP} \downarrow S | S} & \quad \frac{\text{NP} \downarrow S | S}{\text{NP} \downarrow S | S} & \quad \frac{\text{NP} \downarrow S | S}{\text{NP} \downarrow S | S} \\
\text{some} & \quad \phi & \quad \text{remained out} = \quad \text{some remained out} & \quad \text{Lower} \\
\lambda P : \exists X | X \geq 1 \land P(X) \downarrow \text{remained out} & \quad Q & \quad \lambda Q : \exists Y | Y \geq 1 \land Q(Y) \downarrow \text{remained out} \\
\exists X | X \geq 1 \land P(X) \downarrow \text{remained out} \quad \lambda Q : \exists Y | Y \geq 1 \land Q(Y) \downarrow \text{remained out} \\
\lambda Q : \exists X | X \geq 1 \land \text{men} & \quad \lambda Q : \exists Y | Y \geq 1 \land \text{men} \land \text{remained out} \quad \text{Lower} \\
\exists X | X \geq 1 \land \text{men} \land \text{remained out} & \quad \exists Y | Y \geq 1 \land \text{men} \land \text{remained out} \\
\lambda Q : \exists X | X \geq 1 \land \text{men} \land \text{remained out} & \quad \lambda Q : \exists Y | Y \geq 1 \land \text{men} \land \text{remained out} \quad \text{Lower} \\
\exists X | X \geq 1 \land \text{men} \land \text{remained out} & \quad \exists Y | Y \geq 1 \land \text{men} \land \text{remained out} \\
\end{align*}
\]

a fair approximation of the intended meaning: there is a set \( X \) of men who entered and there is a set \( Y \) of men who remained out.

The more (syntactically) complex case which involves the preposition \( \text{of} \) may be treated similarly, but with an important difference: in "most of the men who entered", the quantificational determiner takes a DP as an argument, and not a plural common noun as in "some men". Thus, quantificational determiners should have two distinct lexical entries, one for the case without the preposition \( \text{of} \), that requires a plural common noun as an argument and one for the case with the preposition \( \text{of} \), that requires a plural DP as an argument. We give here the lexical entry for the \( \text{of} \)-variant for the quantificational determiner most, that we need to derive our ellipsis example:
The void lexical entry for DP is:

\[
\frac{D_{\text{DP}} \triangleright S|S}{D_{\text{DP}}} \\
\phi \\
\frac{\lambda Q.[]}{Q}
\]  

(28)

Here is the derivation for \textit{Some men entered. Most (of the men who entered) sat down}:

\[
\frac{D_{\text{DP}} \triangleright S|S}{D_{\text{DP}}} \quad \frac{D_{\text{DP}} \triangleright S|S}{D_{\text{DP}}} \quad \frac{S|S}{D_{\text{DP}}}|S}
\]

\[
\begin{align*}
\text{most of} & \\
\lambda Y. \exists X. X \in Y \wedge |X| \geq |Y| & \phi & \text{sat down} = \\
& \lambda Q.[] & [] & [] & \text{sat down}
\end{align*}
\]

(29)

\[
\frac{D_{\text{DP}} \triangleright S|S}{S} \quad \frac{D_{\text{DP}} \triangleright S|S}{S} \quad \frac{S|S}{S}
\]

\[
\begin{align*}
\text{most of sat down} & \\
\lambda Q.[] & \lambda Q.[] & \text{sat down}
\end{align*}
\]

(30)

\[
\frac{S|D_{\text{DP}} \triangleright S}{S} \quad \frac{S|D_{\text{DP}} \triangleright S}{D_{\text{DP}}}|S|S}
\]

\[
\begin{align*}
X = \text{arg max}_Y & \{Y : \text{men } Y \wedge \text{entered } Y\} & \exists Z. Z \subseteq Q \wedge |Z| \geq |Q| & \text{sat down } Z
\end{align*}
\]

(31)

\[
\frac{S|D_{\text{DP}} \triangleright S}{S} \quad \frac{S|D_{\text{DP}} \triangleright S}{S} \quad \frac{S|S}{S}
\]

\[
\begin{align*}
X = \text{arg max}_Y & \{Y : \text{men } Y \wedge \text{entered } Y\} & \exists Z. Z \subseteq Q \wedge |Z| \geq |Q| & \text{sat down } Z
\end{align*}
\]

(32)

\[
\begin{align*}
\exists X. |X| \geq 1 & \wedge \lambda Q.[]X = \text{arg max}_Y & \{Y : \text{men } Y \wedge \text{entered } Y\} & \wedge \exists Z. Z \subseteq Q \wedge |Z| \geq |Q| & \text{sat down } Z
\end{align*}
\]  

(33)

which means that there is a maximal set $X$ of all men who entered and there is a subset $Z$ of $X$ of cardinality more than half of the cardinality of $X$ and $Z$ sat down.
5. Accommodation

Anaphoric expressions may not always find a suitable antecedent in the previous discourse, (a situation described as *it came out of the blue*). Nevertheless, they are perfectly understood and acceptable, because the reference of that expression is (pragmatically) particularly salient. They are said to be "accommodated". For instance, there are plenty of legal uses of definites such as *the door or his mother* that lack a previously introduced antecedent into the discourse. For example, the sentence *The door closed* is not usually preceded by *There is a door*. In order to account for these facts, we may allow the definite article *the* to introduce variables that do not look left for a binder to transmit a value to them. Thus, we give to the definite article *the*, two alternative lexical entries, one for its regular anaphoric use and one for its accommodated use, respectively:

\[
\begin{align*}
DP^p \rightarrow S|S/N^l \quad \text{and} \quad S|S \rightarrow DP/N \\
\lambda P. \frac{\lambda y. P(y) \langle \| \rangle}{y} \quad \lambda P. \frac{\lambda y. P(y) \langle \| \rangle}{y}
\end{align*}
\]

The above lexical entries stipulate that the regular anaphoric use of *the* imposes a left search for a binder of the newly introduced variable *y*, while the accommodated use gives the new variable existential force (by default, *y* is bounded by the existential quantifier).

The plural version of the definite article receives a similar treatment: one lexical entry for its regular anaphoric use (as in *Some kids came. The kids played.*), and one for its accommodated use (like in *The doors are opened.*):

\[
\begin{align*}
DP^p \rightarrow S|S/N^l \quad \text{and} \quad S|S \rightarrow DP/N \\
\lambda P. \frac{\lambda Y. P(Y) \langle \| \rangle}{Y} \quad \lambda P. \frac{\lambda Y. P(Y) \langle \| \rangle}{Y}
\end{align*}
\]

Another related phenomena is cataphora, i.e. pronominal anaphora that precedes the DP it refers to, like in: *If he comes, John brings wine*. Note that cataphora only occurs at sentential level. At the expense of dramatically raising the number of possible derivations, we could allow pronouns to look right for binders and binders to look left for expressions to bind. The much higher cost is explained by the natural left to right order of natural language processing which cataphora brakes. Another solution is given in (de Groote 2010), where the author chooses to change the role of the definite phrase and of the pronoun: the pronoun introduces an underspecified referent that binds the subsequent definite phrase.

6. Focus

A treatment of focus within the continuation semantics framework was sketched in Barker (2004). He uses the operators *fcontrol* and *run* to account for the semantics of focus. Instead, we will give focus a continuation based interpretation with no such operators. This interpretation only uses the type shifters *Lift* and *Lower*. We will also treat the phenomena of free-focus and bound-focus anaphora, which, not surprisingly, need no further stipulations.

Most (probably all) languages provide some way of marking some constituent in a sentence as having extra prominence. In spoken English, this is typically accomplished in part by a local maximum in the fundamental frequency (the lowest frequency at which the vocal folds are vibrating). By convention, the location of such a 'pitch accent' is indicated typographically by setting the most affected word in capital letters:

1. *JOHN saw Mary.*
2. *John SAW Mary.*
3. *John saw MARY.*
There is a distinct but elusive difference in meaning between these sentences that depends on the location of the pitch accent. In each case, it remains true that John saw Mary, but the piece of information which is being emphasized differs. In traditional terms, the constituent containing the pitch accents is said to be ‘in focus’, which means (very roughly) that it carries the new information provided by the sentence. These observations can be sharpened by noting that the location of the pitch accent correlates with the use of only\(^2\) and with the precise piece of information requested by a question.

1. Who saw Mary?

2. What did John do to Mary?

3. Who did John see?

We will give the focus maker (operator) F the following denotation:

\[
\frac{S|S}{\mathcal{A}} \quad F \quad \lambda x. \frac{\lambda k. k(x) \land \forall y. (y = x \lor \neg (k(y))) (\lambda z. [\_])}{z}
\]

(36)

The type is polymorphic: \(\mathcal{A}\) may be any category. The variable \(z\) is just a means to distribute the context of the focused word to its positive contribution (i.e. it is true what is said about that constituent) and to the negative part (no other relevant choice of constituents of the same type makes the statement true). For instance, here there are the derivations of the upper three examples:

\[
\frac{S|S}{DP} \quad F \quad \lambda x. \frac{\lambda k. k(x) \land \forall y. (y = x \lor \neg (k(y))) (\lambda z. [\_])}{z}
\]

\[
\frac{S|S}{DP} \quad DP \quad \frac{S|S}{(DP, S)/DP} \quad \frac{S|S}{DP} \quad F \quad \lambda x. \frac{\lambda k. k(x) \land \forall y. (y = x \lor \neg (k(y))) (\lambda z. [\_])}{z}
\]

\[
\frac{S|S}{DP} \quad \frac{S|S}{(DP, S)/DP} \quad \frac{S|S}{DP} \quad F \quad \lambda x. \frac{\lambda k. k(x) \land \forall y. (y = x \lor \neg (k(y))) (\lambda z. [\_])}{z}
\]

\[
\frac{S|S}{S} \quad F \quad \lambda x. \frac{\lambda k. k(x) \land \forall y. (y = x \lor \neg (k(y))) (\lambda z. [\_])}{z}
\]

\[
\frac{S|S}{S} \quad \frac{S|S}{(DP, S)/DP} \quad \frac{S|S}{DP} \quad F \quad \lambda x. \frac{\lambda k. k(x) \land \forall y. (y = x \lor \neg (k(y))) (\lambda z. [\_])}{z}
\]

(37)

(38)

(39)

The use of “only” enhances the focus semantics, making it more obvious. It appears that focus placement can affect grammaticality. Jackendoff (1972) noted:

1. John only gave his DAUGHTER a new bicycle.

2. *JOHN only gave his daughter a new bicycle.

The apparent generalization is that only must have a focus within its scope. Focus also interacts in an interesting way with ellipsis, for instance as in:

JOHN loves Mary. Not Sue. (=not Sue loves Mary).
John loves MARY. Not Sue. (=not John loves Sue).

We leave these issues for further research in the continuation semantics framework.
\[
\begin{align*}
F \text{ John saw Mary} &= F \text{ John saw Mary} \\
(\lambda z. (\text{saw } m \ z(j)) \land \forall y.(y = j \lor \neg(\lambda z. \text{saw } m \ z(y)))) & \quad \text{saw } m \ j \land \forall y.(y = j \lor \neg(\text{saw } m \ y))
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
John \ F \text{ saw Mary} & \quad \text{Lower} \\
(\lambda k. (\text{saw } j(y)) \land \forall y.(y = m \lor \neg(\lambda z. \text{saw } j(y)))) & \quad (\lambda k. (\text{saw } j(y)) \land \forall y.(y = m \lor \neg(\lambda z. \text{saw } j(y))))
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
F \text{ saw Mary} & \quad \text{Lower} \\
(\lambda k. (\text{saw } j(y)) \land \forall y.(y = m \lor \neg(\lambda z. \text{saw } j(y)))) & \quad (\lambda k. (\text{saw } j(y)) \land \forall y.(y = m \lor \neg(\lambda z. \text{saw } j(y))))
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]

\[
\begin{align*}
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP \\
\frac{\frac{\frac{S | S}{DP}}{(DP \backslash S)/DP} / ((DP \backslash S)/DP)}{(DP \backslash S)/DP} & \quad (DP \backslash S)/DP
\end{align*}
\]
There is no point in using continuations if the context of the focused word is as simple as it is in the upper examples. But the context may be of any complexity, for example as in:

Mary tried to dance with F(JOHN).
Mary tried to F(DANCE) with John.

There is still a problem with the given focus interpretation: the universal quantifier in the upper denotation quantifies over absolutely every semantic objects of the focused word type. But this is a too strong interpretation. The standard assumption is that the quantification over alternatives is context dependent. The quantifier should only quantify over the contextually relevant objects. For instance, "F(John) saw Mary," does not mean that absolutely nobody saw Mary (except John), but that no people from the relevant context saw Mary. We assume that what ought to count as a contextually-relevant object is a pragmatic issue and not a semantic one.

Now, we give an account within this framework of the notions of free-focus and bound-focus anaphora. The sentence "F(John) thinks he is smart." is ambiguous between the following two meanings:

- John is the only individual y who thinks John (j) is smart
- John is the only individual y who thinks y is smart.

The first reading is called free-focus and the second one bound-focus. We give the two derivations that show how the two different meanings are obtained:

\[
\begin{aligned}
S_j & \quad S_{DP\triangleright S} \\
\frac{\text{DP}}{DP} & \quad \frac{\text{S}_j}{S} \\
F & \quad \text{John} \\
\lambda x. [\lambda k. k(j) \land \forall y.(y = j \lor \neg(k(y)))(\lambda z. [\text{think (is smart) z}])j & \quad \lambda x. [\lambda k. k(y) \land \forall y.(y = j \lor \neg(\lambda z. [\text{think (is smart) z}](y)))]j
\end{aligned}
\]

\[
\begin{aligned}
S_j & \quad S_{DP\triangleright S} \\
\frac{\text{DP}}{DP} & \quad \frac{\text{S}_j}{S} \\
F & \quad \text{John} \\
\lambda x. [\lambda k. k(j) \land \forall y.(y = j \lor \neg((\lambda z. [\text{think (is smart) z}])y))]j & \quad \lambda x. [\lambda k. k(j) \land \forall y.(y = j \lor \neg(\lambda z. [\text{think (is smart) z}](y))]j
\end{aligned}
\]

\[
\begin{aligned}
S_j & \quad S_{DP\triangleright S} \\
\frac{\text{DP}}{DP} & \quad \frac{\text{S}_j}{S} \\
F & \quad \text{John} \\
\lambda x. [\lambda k. k(j) \land \forall y.(y = j \lor \neg(\lambda z. [\text{think (is smart) z}](y))]j & \quad \lambda x. [\lambda k. k(j) \land \forall y.(y = j \lor \neg(\lambda z. [\text{think (is smart) z}](y))]j
\end{aligned}
\]

\[
\begin{aligned}
S_j & \quad S_{DP\triangleright S} \\
\frac{\text{DP}}{DP} & \quad \frac{\text{S}_j}{S} \\
F & \quad \text{John} \\
\lambda x. [\lambda k. k(j) \land \forall y.(y = j \lor \neg(\lambda z. [\text{think (is smart) z}](y))]j & \quad \lambda x. [\lambda k. k(j) \land \forall y.(y = j \lor \neg(\lambda z. [\text{think (is smart) z}](y))]j
\end{aligned}
\]
\[ \lambda x. (\lambda k.(x) \wedge \forall y.(y = x \lor \neg (k(y))))(\lambda z.[\ ])(\lambda t.[\ ])(\lambda t.[\ ]z) \]

\[ \lambda \bar{x}. (\lambda k.(\bar{x}) \wedge \forall y.(y = \bar{x} \lor \neg (k(y))))(\lambda z.[\ ])(\lambda t.[\ ])(\lambda t.[\ ]z) \]
7. Conclusions

We gave in this paper an explicit formal account of discourse semantics that extends Barker and Shan’s (2008) (sentential) semantics based on continuations. We shifted from sentential level to discourse level. In this framework, we showed how continuation-based discourse semantics may account for linguistic phenomena such as aspects of hierarchical discourse structure, ellipses, accommodation and free-focus versus bound-focus anaphora. Moreover, there was no need for extra stipulations in order to give a compositional interpretation to these phenomena. This is not surprising, due to the unified account of scope-taking provided by continuation semantics.

No other theory to our knowledge allows quantifiers and anaphora to interact in a uniform system of scope taking, in which quantification and binding employ the same mechanism. We argue that continuations are a versatile and powerful tool to be used not only in computer science, but also in linguistics in general and in natural language semantics in particular.

References


